quadratische Ergänzung

1) $x^2 + px + q = 0$ $\qquad (a+b)^2 = a^2 + 2ab + b^2$

$x^2 + 4x + 1 = (x+2)^2 - 4 + 1 = (x+2)^2 - 3$

$x^2 + px + q = \left(x + \frac{p}{2}\right)^2 - \left(\frac{p}{2}\right)^2 + q \stackrel{!}{=} 0$

$\Longleftrightarrow \quad \left(x + \frac{p}{2}\right) = \pm\sqrt{\left(\frac{p}{2}\right)^2 - q}$

$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$

2) geometrische Reihe

$\sum_{n=0}^{N} q^n = 1 + q + q^2 + \dots q^N$

$q \cdot \sum_{n=0}^{N} q^n = q + q^2 + q^3 + \quad q^{N+1}$

$\sum_{n=0}^{N} q^n - q\sum_{n=0}^{N} q^n = 1 - q^{N+1}$

$\sum_{n=0}^{N} q^n (1-q) = 1 - q^{N+1}$

$\sum_{n=0}^{N} q^n = \frac{1-q^{N+1}}{1-q} \qquad \square$

Grenzübergang $n \to \infty$

$\sum_{n=0}^{\infty} q^n = \frac{1-q^{N+1}}{1-q} = \frac{1}{1-q}$ $\qquad q^{N+1} \to 0$

für $|q| < 1$

Einheitskreis und aufmalen

3) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

b) $2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha$

$\sin^2\frac{\alpha}{2} + \underbrace{\sin^2\frac{\alpha}{2} - \cos\alpha}_{\cos^2\frac{\alpha}{2}} = 1$

$\sin^2\frac{\alpha}{2} - \cos\alpha \stackrel{!}{=} \cos^2\frac{\alpha}{2}$

$\sin^2\alpha - \cos^2\alpha = \cos 2\alpha$ ✓

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1}{2}(1+\cos\alpha)}$$

$$2\cos^2\frac{\alpha}{2} = 1+\cos\alpha$$

$$\cos^2\alpha + \cos^2\alpha = 1 + \cos 2\alpha$$

$$\cos^2\alpha + \underbrace{\cos^2\alpha - \cos 2\alpha}_{\sin^2\alpha} = 1$$

$$\cos^2\alpha - \cos 2\alpha = \sin^2\alpha$$

$$\cos^2\alpha - \sin^2\alpha = \cos(\alpha+\alpha) \quad ✓$$

(c) $\frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] = \frac{1}{2}\left[\cancel{\cos\alpha\cos\beta} + \sin\alpha\sin\beta\right.$

$\left. - (\cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta)\right]$ ✓

$\frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right] = \frac{1}{2}\left[\sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta}\right.$

$\left. + \sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta}\right]$

(d) $\sin 3\alpha = \sin(2\alpha+\alpha) = \sin 2\alpha\cos\alpha + \cos 2\alpha\sin\alpha$

~~= sinα cosα + cosα sinα~~

$= 2\sin\alpha\cos\alpha\cos\alpha + (\cos^2\alpha - \sin^2\alpha)\sin\alpha$

$= 2\sin\alpha\cos^2\alpha + \cos^2\sin - \sin^3$

$= \cos^2(2\sin + \sin) - \sin^3$

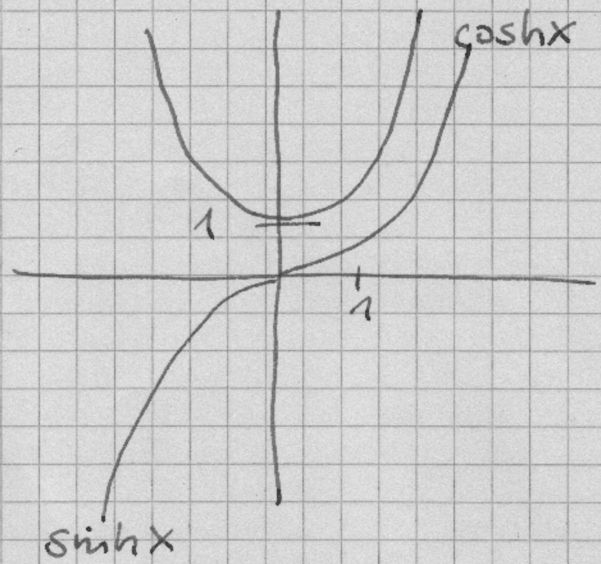
$= \cos^2 3\sin - \sin^3$

$= (1-\sin^2)\,3\sin - \sin^3$

(e) $$\frac{\cos\alpha\,(1-\tan^2\alpha)}{\tan\alpha\,(\underbrace{\cot\alpha}_{\frac{1}{\tan\alpha}}-1)} = \frac{\cos\alpha\,(1+\tan\alpha)\cancel{(1-\tan\alpha)}}{\cancel{1-\tan\alpha}}$$

4) $\sinh x = \frac{1}{2}(e^{x} - e^{-x})$ $\qquad \tanh x = \frac{\sinh}{\cosh}$

$\cosh x = \frac{1}{2}(e^{x} + e^{-x})$ $\qquad \coth x = \frac{\cosh}{\sinh}$



(a) $\cosh^2 x - \sinh^2 x$

$$= \frac{1}{4}(e^{x}+e^{-x})^2 - \frac{1}{4}(e^{x}-e^{-x})^2$$

$$= \frac{1}{4}\left(\cancel{e^{2x}} + 2e^{x}e^{-x} + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2e^{x}e^{-x} - \cancel{e^{-2x}}\right) = \frac{4}{4}$$

(b) $\sinh(x+y) = \frac{1}{2}(e^{x+y} - e^{-x-y}) = \frac{1}{2}(e^{x}e^{y} - e^{-x}e^{-y})$

$$= \frac{1}{2}\Big[(\cosh x + \sinh x)(\cosh y + \sinh y) - (\cosh x - \sinh x)(\cosh y - \sinh y)\Big]$$

gemischte doppelt, quadrate fallen raus

$$= \frac{1}{2}\Big[2\cosh x \sinh y + 2\sinh x \cosh y\Big]$$

$$= \cosh x \sinh y + \sinh x \cosh y$$

(c) $\cosh(x+y) = \frac{1}{2}(e^{x+y} + e^{-x-y}) = \frac{1}{2}(e^{x}e^{y} + e^{-x}e^{-y})$

$$= \frac{1}{2}\Big[(\cosh x + \sinh x)(\cosh y + \sinh y) + (\cosh x - \sinh x)(\cosh y - \sinh y)\Big]$$

$$= \frac{1}{2}\Big[2\cosh x \cosh y + 2\sinh x \sinh y\Big]$$

$$= \cosh x \cosh y + \sinh x \sinh y$$

5) Parabel

$f(x) = ax^2 + \cancel{bx} + c$

$f(-5) = 0 \qquad f(5) = 0 \qquad f(0) = 4$

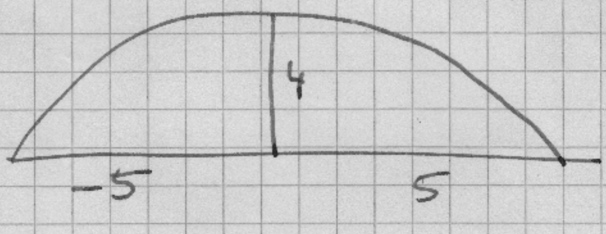
$f(x) = \cancel{a}\, A\,(x - x_1)(x - x_2)$

$A\,(x-5)(x+5)$

$A\,(-5)(+5) = 4 \qquad A = -\frac{4}{25}$

$-\frac{4}{25}(x-5)(x+5) = -\frac{4}{25}x^2 + \frac{4}{\cancel{25}}\,\cancel{25}$

$= -\frac{4}{25}x^2 + 4$



8) Erde $R = 6\ 378\ 137$ m

$U = 2\pi R = 40\ 075\ 016$ m

$U_{neu} = 40\ 075\ 017$ m

$R_{neu} = 6\ 378\ 137{,}16$ m

steht 16 cm ab

Gürtel $U = 2\pi R = 90$ cm $= 0{,}90$ m

$R = 0{,}14$ m $= 14$ cm

$U_{neu} = 1{,}90$ m

$R_{neu} = 0{,}30$ m

steht 16 cm ab nämlich $\frac{1\,m}{2\pi}$

$$U + x = 2\pi R + x = 2\pi \left(R + \frac{x}{2\pi} \right)$$

6) Polynomdivision

$$(x^2 - 2x + 3) : (x^2 + 1) = \underline{\underline{1 + \frac{-2}{x} + \frac{2}{x^2} + O\left(\frac{1}{x^3}\right)}}$$

$\underline{x^2 + 1}$

$-2x + 2$

$\underline{-2x - \frac{2}{x}}$

$2 + \frac{2}{x}$

$\underline{2 + \frac{2}{x^2}}$

$\frac{2}{x} - \frac{2}{x^2}$

$$(x^3 - 3x^2 + 4x + 1) : (x^2 + 2) = \underline{\underline{x - 3 + \frac{2}{x} + \frac{7}{x^2} + O\left(\frac{1}{x^3}\right)}}$$

$\underline{x^3 + 2x}$

$-3x^2 + 2x + 1$

$\underline{-3x^2 - 6}$

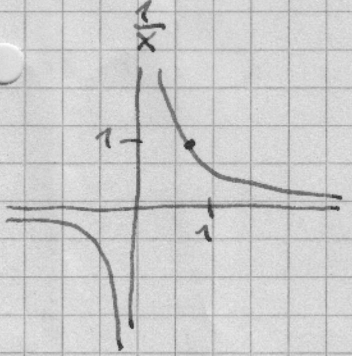
$2x + 7$

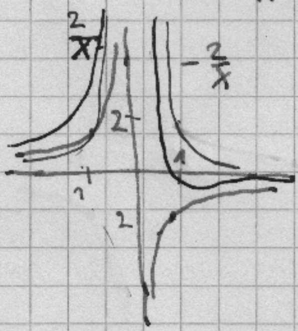
$\underline{2x + \frac{4}{x}}$

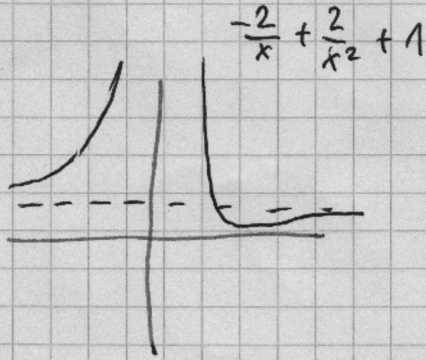
$7 - \frac{4}{x}$

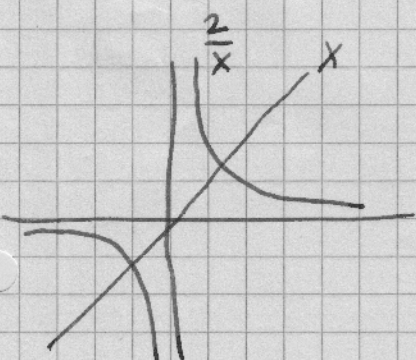
$\underline{7 + \frac{14}{x^2}}$

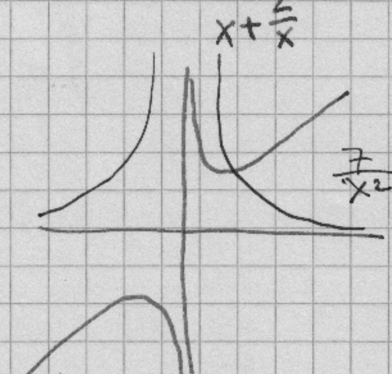
$-\frac{4}{x} - \frac{14}{x^2}$

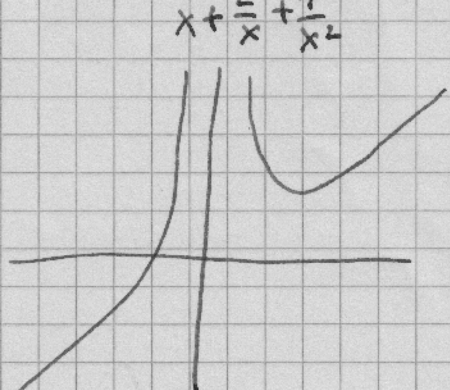












7) a) 1.

$$\underbrace{\overset{N}{\_\_}\ \overset{N-1}{\_\_}\ \overset{N-2}{\_\_}\ \_\_\ \overset{N-k}{\_\_}}_{k}$$

$$\frac{N!}{(N-k)!\quad k!} = \binom{N}{k}$$

2.

$$\underbrace{\overset{(1-p)}{\_}\ \overset{p}{\underline{\times}}\ \overset{(1-p)}{\_}\ \overset{p}{\underline{\times}}\ \overset{(1-p)}{\_}}_{N}$$

$$p^k\ (1-p)^{N-k}$$

$$\underline{\times}\ \_\ \_\ \_\ \underline{\times}$$

mögliche Realisierungen

= # Möglichkeiten k auf N Plätze zu verteilen

= # Möglichkeiten k aus N zu ziehen $= \binom{N}{k}$

$$P(k) = \binom{N}{k}\ p^k \cdot (1-p)^{N-k}$$

3. $$\binom{N}{k} = \binom{N}{N-k}$$

Möglichkeiten k aus N zu ziehen

= # Möglichkeiten genau die $(N-k)$ in der Urne liegen zu lassen

$$\sum_{k=0}^{N} \binom{N}{k}\ p^k (1-p)^{N-k} = 1 \quad \text{für } 0<p<1$$

Wahrscheinlichkeiten von 1 bis N mal Kopf zu erhalten, sind alle möglichen Ausgänge des Experiments. Müssen zusammen 1 sein.

(b) N Kugeln

k mal Erfolg = Weiße

l Elemente der Stichprobe

m Erfolge in der Stichprobe

$\underline{P}(k)$

$$= \frac{\binom{k}{m}\binom{N-k}{l-m}}{\binom{N}{l}}$$

$$= \frac{\left(\begin{array}{c}\text{aus } k \text{ Erfolgen} \\ m \text{ Erfolge in der Stichprobe} \\ \text{zu } \cancel{\text{ziehen}} \text{ ziehen}\end{array}\right)\left(\begin{array}{c}\text{aus } N-k \text{ Mißerfolgen} \\ l-m \text{ Mißerfolge in der} \\ \text{Stichprobe zu ziehen}\end{array}\right)}{\left(\begin{array}{c}\text{aus } N \text{ Kugeln} \\ l \text{ ziehen}\end{array}\right)}$$