Differenzieren / Kurvendiskussion

Lösung Aufgabe 1:

(a) $f'(x) = 4(1-3x^2)^3 \cdot (-6x) = -24x(1-3x^2)^3$

(b) $f'(x) = \dfrac{2x-5}{2\sqrt{x^2-5x+2}}$

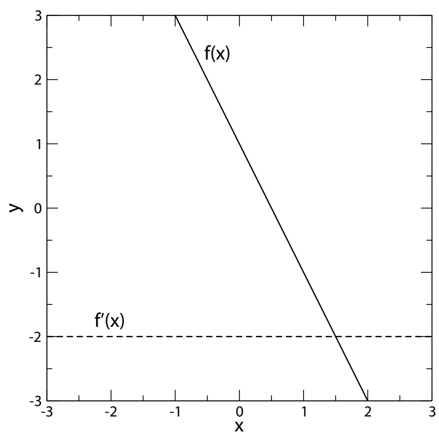
(c) $f'(x) = \dfrac{-\sqrt{x} - (1-x) \cdot \frac{1}{2\sqrt{x}}}{x} = \dfrac{-2x-(1-x)}{2x^{3/2}} = -\dfrac{1+x}{2x^{3/2}}$

(d) $f'(x) = \dfrac{1}{2}\sqrt{\dfrac{x+1}{x-1}} \cdot \dfrac{(x+1)-(x-1)}{(x+1)^2} = \dfrac{1}{\sqrt{x-1} \cdot (x+1)^{3/2}}$

(e) $f'(x) = -\dfrac{1}{(\sqrt{x}-1)^2} \cdot \dfrac{1}{2\sqrt{x}} = -\dfrac{1}{2\sqrt{x}(\sqrt{x}-1)^2}$

Lösung Aufgabe 2:

(a) Nullstelle $x_0 = \dfrac{1}{2}$; y-Achsenabschnitt: $f(0) = 1; f'(x) = -2$

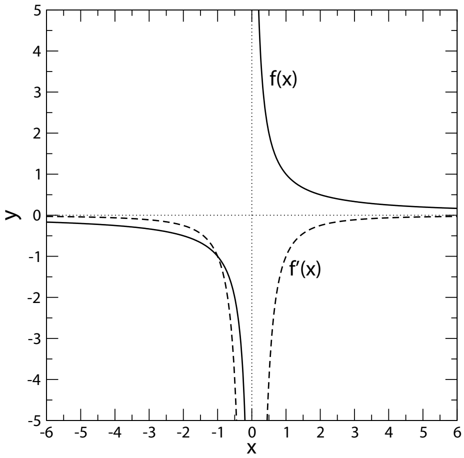


(b) $f(x) = \dfrac{1}{x}$ $\qquad f(x) \underset{x\to\pm\infty}{\longrightarrow} 0$

$f(x) \underset{x\to\pm 0}{\longrightarrow} \pm\infty$

$f'(x) = -\dfrac{1}{x^2}$ $\qquad f'(x) \underset{x\to\pm\infty}{\longrightarrow} 0$

$f'(x) \underset{x\to\pm 0}{\longrightarrow} -\infty$



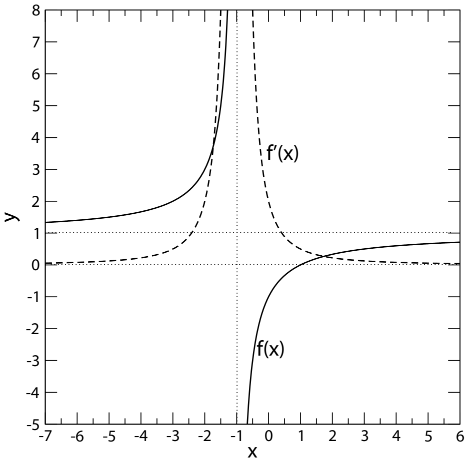
(c) $f(x) = \dfrac{x-1}{x+1}$

$f(x) \underset{x\to\pm\infty}{\longrightarrow} 1$

$f(x) \underset{x\to -1\pm 0}{\longrightarrow} \pm\infty$

$f(x) = 0 \qquad \text{für} \qquad x = 1$

$f(0) = -1$



$$f'(x) = \frac{x+1-(x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$f'(x) \underset{x\to -1\pm 0}{\longrightarrow} \infty$

$f'(x) \underset{x\to\infty}{\longrightarrow} 0$

$f(0) = 2$

Lösung Aufgabe 3:

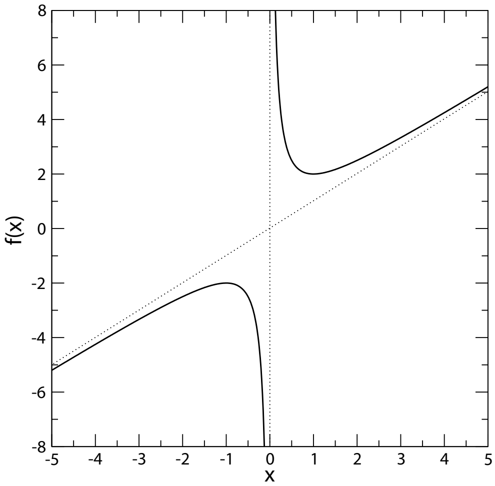
(a) $f'(x) = 1 - \dfrac{1}{x^2} \qquad f'(x) = 0 \qquad \text{für} \qquad x = \pm 1$

$f''(x) = \dfrac{2}{x^3} \qquad \rightarrow \qquad x = +1 \quad$ ist Minimum

$x = -1 \quad$ ist Maximum

$f(x) \sim x \qquad \text{für } x \to \pm\infty$

$f(x) \underset{x\to\pm 0}{\longrightarrow} \pm\infty$

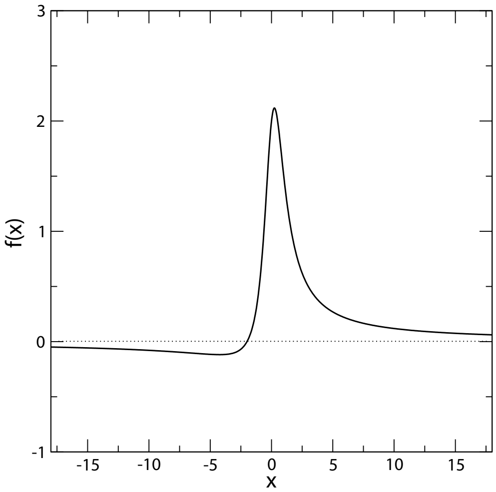


(b) $f'(x) = \frac{x^2+1-(x+2)2x}{(x^2+1)^2} = \frac{-x^2-4x+1}{(x^2+1)^2}$

$\rightarrow\ f'(x) = 0 \quad \text{wenn} \quad x^2+4x-1=0 \ \Rightarrow\ x_{1/2} = -2 \pm \sqrt{4+1} = -2 \pm \sqrt{5}$

$f(x) \underset{x\to\pm\infty}{\longrightarrow} 0$

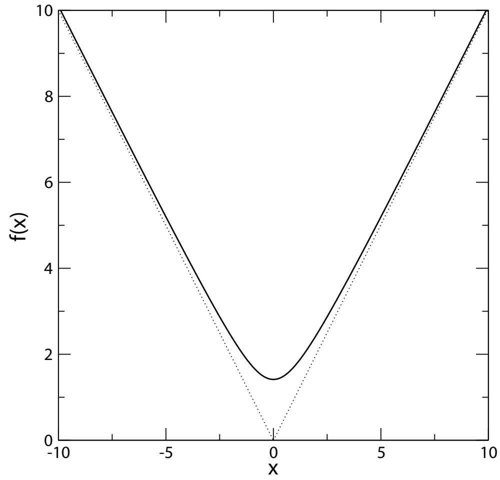
$f(x) = 0$ wenn $x = -2$



(c) $f'(x) = \frac{2x}{2\sqrt{x^2+2}} = \frac{x}{\sqrt{x^2+2}} \Rightarrow\ f'(x) = 0$ für $x = 0$

$f(x) \underset{x\to\infty}{\longrightarrow} |x|$

$f(0) = \sqrt{2}$



Lösung Aufgabe 4:

(a) $f'(x) = -\frac{2x}{(x^2+a^2)^2}$

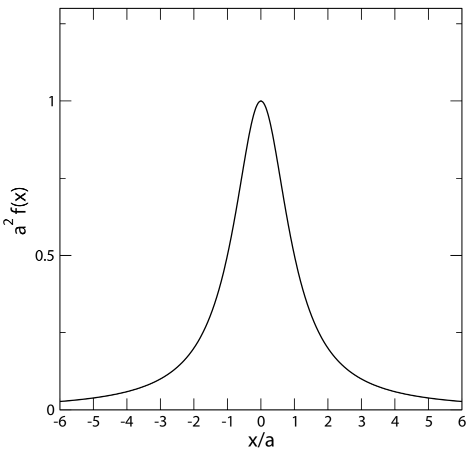
$$f''(x) = -\frac{2(x^2+a^2)^2 - 2x \cdot 2(x^2+a^2) \cdot 2x}{(x^2+a^2)^4}$$

$$= -\frac{2(x^2+a^2) - 8x^2}{(x^2+a^2)^4}$$

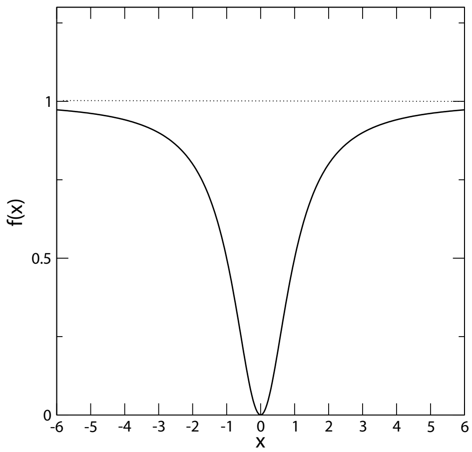
$\Rightarrow\ f''(x) = 0$ wenn $2a^2 - 6x^2 = 0 \ \Rightarrow\ x = \pm\frac{a}{\sqrt{3}}$

$f(x) \underset{x\to\pm\infty}{\longrightarrow} 0$

$f(0) = \frac{1}{a^2}$



(b) $f(x) = 1 - \frac{1}{x^2+1}$ $\rightarrow$ bis auf Verschiebung in y und Vorzeichen wie in (a) mit $a = 1$



Lösung Aufgabe 5:

(a) $f'(x) = 2x\ \cos(x^2)$

(b) $f'(x) = 2\sin x\ \cos x$

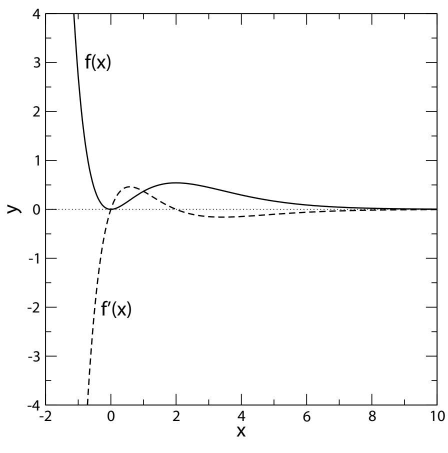
(c) $f'(x) = \cos\left[a\ \cos(bx)\right] \cdot (-a)\sin(bx) \cdot b$
$\quad = -ab\ \sin bx\ \cos\left[a\ \cos(bx)\right]$

Lösung Aufgabe 6:

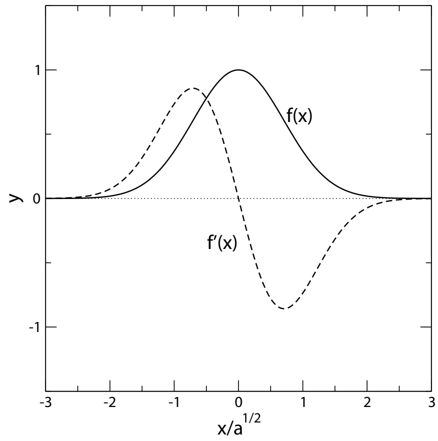
(a) $f(x) \underset{x\to+\infty}{\longrightarrow} 0 \qquad f(x) \underset{x\to-\infty}{\longrightarrow} \infty \qquad f(0) = 0$

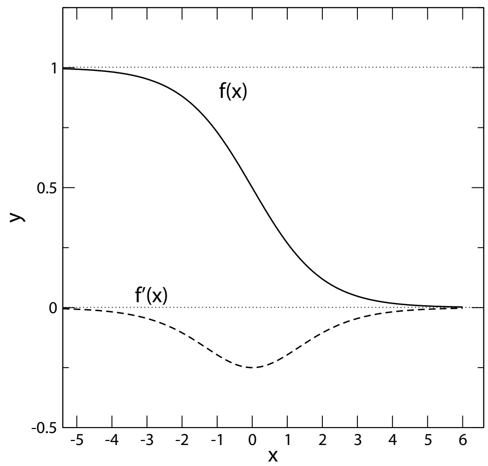
$f'(x) = -x^2\, e^{-x} + 2xe^{-x} = -x(x-2)e^{-x}$

$\rightarrow\ f'(x) = 0$ für $x = 0$ und $x = 2$



(b) $f'(x) = -2ax\, e^{-ax^2}$





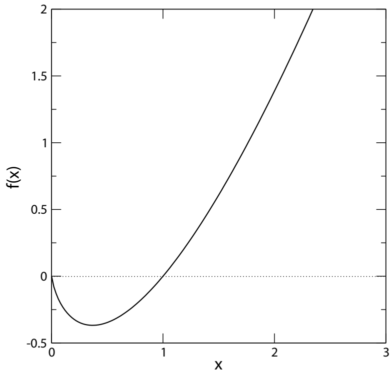
(c)

(Lösung von (b) und (c) nur noch sehr schematisch)

Lösung Aufgabe 7:

(a) $f(0) = 0\,;\; f'(x) = \ln x + 1 \;\Rightarrow\; f'(x) = 0$ für $x = \dfrac{1}{e}$

$f''(x) = \dfrac{1}{x} \neq 0\,.$ $\qquad f(x) = 0 \qquad$ für $x = 0$ und $x = 1$



(b) $f(0) = -\infty \qquad f(x) = 0$ für $x = (x+1)^2$

$$\Rightarrow\; x^2 + x + 1 = 0$$

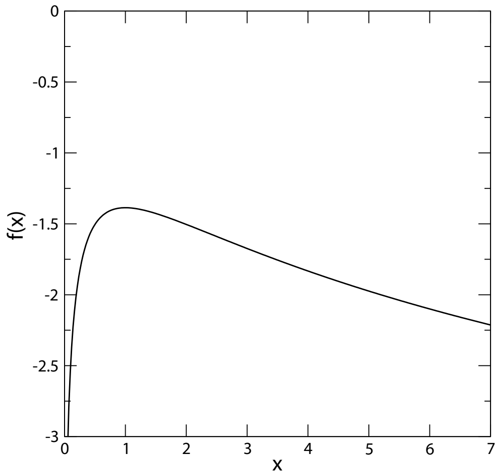
$$\Rightarrow\; x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1}$$

$$\rightarrow \text{ keine Nullstellen}$$

$$f'(x) = \frac{(x+1)^2}{x} \cdot \frac{(x+1)^2 - x2(x+1)}{(x+1)^4} = \frac{x+1-2x}{x(x+1)} = -\frac{x-1}{x(x+1)}$$

$\Rightarrow \; f'(x) = 0$ für $x = 1$.

$f(x) \sim \ln \frac{1}{x} = -\ln x \quad$ für $x \to \infty$



Lösung Aufgabe 8:

$\sinh(\operatorname{arsinh} x) = x$

$\Rightarrow \; \cosh(\operatorname{arsinh} x) \cdot (\operatorname{arsinh} x)' = 1$

$$\Rightarrow \; (\operatorname{arsinh} x)' = \frac{1}{\cosh(\operatorname{arsinh} x)}$$

$$= \frac{1}{\sqrt{1 + \sinh^2(\operatorname{arsinh} x)}} = \frac{1}{\sqrt{1+x^2}} \qquad .$$

$$\rightarrow \; (\operatorname{arsinh} x)' = \frac{1}{\sqrt{1+x^2}}$$

$\cosh(\operatorname{arcosh} x) = x$

$\Rightarrow \; \sinh(\operatorname{arcosh} x)(\operatorname{arcosh} x)' = 1$

$$\Rightarrow \; (\operatorname{arcosh} x)' = \frac{1}{\sinh(\operatorname{arcosh} x)} = \frac{1}{\sqrt{\cosh^2(\operatorname{arcosh} x) - 1}} = \frac{1}{\sqrt{x^2-1}}$$