Aufgabe 1

siehe Blatt 6

Aufgabe 2

siehe Blatt 6

Aufgabe 3

(a)

$$\begin{pmatrix} 8 & -3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 34 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & -1 & 1 \\ -3 & -5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 8 & -3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 34 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3/8 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11/8 \\ 34 \end{pmatrix} \quad \text{erste Zeile } :8$$

$$\begin{pmatrix} 1 & -3/8 \\ 1 & 2/5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11/8 \\ 34/5 \end{pmatrix} \quad \text{zweite Zeile } :5$$

$$\begin{pmatrix} 1 & -3/8 \\ 0 & 31/40 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11/8 \\ 217/40 \end{pmatrix} \quad \text{zweite minus erste Zeile}$$

$$\begin{pmatrix} 1 & -3/8 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11/8 \\ 7 \end{pmatrix} \quad \text{Zweite Zeile } : \frac{31}{40}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} \quad \text{erste Zeil } + \frac{3}{8} \text{ zweite Zeile}$$

$$\Rightarrow \quad x = 4 \quad , y = 7$$

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & -1 & 1 \\ -3 & -5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3/2 & -1/2 \\ 1 & -1 & 1 \\ 1 & 5/3 & -2/3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1/3 \end{pmatrix}$$
erste Zeile $:2$
dritte Zeile $:(-3)$

$$\begin{pmatrix} 1 & 3/2 & -1/2 \\ 0 & -5/2 & 3/2 \\ 0 & 1/6 & -1/6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -4/3 \end{pmatrix}$$
zweite Zeile minus erste Zeile
dritte Zeile minus erste Zeile

$$\begin{pmatrix} 1 & 3/2 & -1/2 \\ 0 & 1 & -3/5 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2/5 \\ -8 \end{pmatrix}$$
zweite Zeile $:(-\frac{5}{2})$
dritte Zeile $:(\frac{1}{6})$

$$\begin{pmatrix} 1 & 3/2 & -1/2 \\ 0 & 1 & -3/5 \\ 0 & 0 & -2/5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2/5 \\ -42/5 \end{pmatrix}$$
dritte minus zweite Zeile

$$\begin{pmatrix} 1 & 3/2 & -1/2 \\ 0 & 1 & -3/5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2/5 \\ 21 \end{pmatrix}$$
dritte Zeile $:(-\frac{2}{5})$

$$\begin{pmatrix} 1 & 3/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 23/2 \\ 13 \\ 21 \end{pmatrix}$$
erste Zeile $+(\frac{1}{2})$ dritte Zeile
zweite Zeile $+(\frac{3}{5})$ dritte Zeile

erste Zeile $-\frac{3}{2}$ zweite Zeile
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8 \\ 13 \\ 21 \end{pmatrix}$$

(c) Winkel α, β, γ

β doppelt so groß wie α $\Rightarrow 2\alpha = \beta \Leftrightarrow 2\alpha - \beta = 0$

α und β zusammen so groß wie $\gamma \Rightarrow \alpha + \beta = \gamma \Leftrightarrow \alpha + \beta - \gamma = 0$

Außerdem $\alpha + \beta + \gamma = 180$ da Dreieck

$\Rightarrow$
$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 180 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & -2 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 180 \\ -360 \\ -180 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 90 \\ -180 \\ 90 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 90 \\ 60 \\ 90 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 30 \\ 60 \\ 90 \end{pmatrix}$$

Aufgabe 4

$3i + 5i = (3+5)i = \underline{\underline{8i}}$ $\qquad -2i + 4i = (-2+4)i = 2i$

$i^2 = i \cdot i = \underline{\underline{-1}}$ $\qquad (-i)^2 = (-1)^2 (i)^2 = (-1)\cdot(-1) = \underline{\underline{1}}$

$1 + (-i)^3 = 1 + (-1)^3 i^3 = 1 + (-1)(i)^2 \cdot i = \underline{\underline{1+i}}$

$1 + (-i)^2 = 1 + i^2 = 1 - 1 = \underline{\underline{0}}$

$i^3 = i^2 \cdot i = \underline{\underline{-i}}$ $\qquad i^4 = (i^2)^2 = (-1)^2 = \underline{\underline{1}}$

$i^5 = (i)^4 \cdot i = \underline{\underline{i}}$ $\qquad i^{12} = (i^2)^6 = (-1)^6 = \underline{\underline{1}}$

$i^{17} = (i^2)^8 i = \underline{\underline{i}}$ $\qquad i^{23} = (i^2)^{11} \cdot i = \underline{\underline{-i}}$

$i^{1125} = (i^2)^{562} \cdot i = \underline{\underline{i}}$

Aufgabe 5

a) $(\lambda_1 z_1 + \lambda_2 z_2)^* = [\lambda_1(x_1 + iy_1) + \lambda_2(x_2 + iy_2)]^*$

$= [\lambda_1 x_1 + \lambda_2 x_2 + i(\lambda_1 y_1 + \lambda_2 y_2)]^*$

$= \lambda_1 x_1 + \lambda_2 x_2 - i(\lambda_1 y_1 + \lambda_2 y_2)$

$= \lambda_1(x_1 - iy_1) + \lambda_2(x_2 - iy_2) = \underline{\underline{\lambda_1 z_1^* + \lambda_2 z_2^*}}$

b)

$(z_1 z_2)^* = [(x_1 + iy_1)\cdot(x_2 + iy_2)]^* = [x_1x_2 + i(x_1y_2 + y_1x_2) - iy_1y_2]^*$

$= x_1x_2 - i(x_1y_2 + y_1x_2) - y_1y_2 = x_1(x_2 - iy_2) - y_1(y_2 + ix_2)$

$= x_1(x_2 - iy_2) - iy_1(+x_2 + \underbrace{\tfrac{1}{i}}_{-i} y_2)$ $\qquad$ NR: $\frac{1}{i} = \frac{i^4}{i} = i \cdot i^2 = -i$

$= x_1(x_2 - iy_2) - iy_1(x_2 - iy_2)$

$= (x_1 - iy_1)(x_2 - iy_2) = \underline{\underline{z_1^* z_2^*}}$

c) $\qquad (z_1^*)^* = [(x_1 + iy_1)^*]^* = [x_1 - iy_1]^* = x_1 + iy_1 = \underline{\underline{z_1}}$

d) $\qquad \left(z_1/z_2\right)^* = \left(\frac{z_1 z_2^*}{|z_2|^2}\right)^* = \frac{z_1^* z_2}{|z_2|^2} = \underline{\underline{\frac{z_1^*}{z_2^*}}}$

c) $\frac{1}{2}(z_1 + z_1^*) = \frac{1}{2}(x_1 + iy_1 + x_1 - iy_1) = \frac{1}{2} 2x_1 = x_1 = \underline{\underline{\mathrm{Re}(z_1)}}$

$\frac{1}{2i}(z_1 - z_1^*) = \frac{1}{2i}(x_1 + iy_1 - (x_1 - iy_1)) = \frac{1}{2i} 2iy_1 = y_1 = \underline{\underline{\mathrm{Im}(z_1)}}$

Aufgabe 6

a) $(3+2i) - (7 + \frac{1}{2}i) = 3-7 + i(2 - \frac{1}{2}) = \underline{\underline{-4 + \frac{3}{2}i}}$

$(-40+i) + (-2+3i) = -40-2 + i(1+3) = \underline{\underline{-42 + 4i}}$

$2i + (-7-2i) = \underline{\underline{-7}}$

b) $(1+i)(1-2i) = 1 + i - 2i - 2i^2 = \underline{\underline{3 - i}}$

$(1+2i)(1-2i) = 1 + 2i - 2i - 4i^2 = \underline{\underline{5}}$

$(3+2i)(-7-\frac{1}{2}i) = -(3+2i)(7+\frac{1}{2}i) = -[21 + 14i + \frac{3}{2}i + i^2]$

$= -[20 + \frac{31}{2}i] = \underline{\underline{-20 - \frac{31}{2}i}}$

$(-40+i)(-2+3i) = 80 - 2i - 120i + 3i^2$

$= \underline{\underline{77 - 122i}}$

c) $(1+i) = \sqrt{2}\, e^{i\frac{\pi}{4}} \Rightarrow (1+i)^4 = (\sqrt{2})^4 (e^{i\frac{\pi}{4}})^4 = 4e^{i\pi} = \underline{\underline{-4}}$

$(1+i)^4 = [(i+1)^2]^2 = [(i+1)(i+1)]^2 = [-1 + i + i + 1]^2$

$= [2i]^2 = \underline{\underline{-4}}$

$(1+i)^{13} = (1+i)^{12}(1+i) = ((1+i)^4)^3 (1+i) = \underbrace{-4^3}_{2^6 = 64}(1+i) = \underline{\underline{-64(i+1)}}$

$(1+i\sqrt{3})^6 = [(1+i\sqrt{3})^2]^3 = [1 + 2i\sqrt{3} - 3]^3 = (-2 + 2i\sqrt{3})^3$

$= (-2)^3 + 6(-2)^2\sqrt{3}i + 3(-2)12i^2 + i^3(2\sqrt{3})^3$

$= -8 + 24\sqrt{3}i + 36 - i\,72\sqrt{3} \neq \underline{64}$

$$\frac{(1+i)^5}{(\sqrt{3}+i)^2} = \frac{(1+i)^4(1+i)}{(3+2\sqrt{3}i-1)} = \frac{-4(1+i)}{2(1+\sqrt{3}i)} = \frac{-2(1+i)(1-\sqrt{3}i)}{2(1+\sqrt{3}i)(1-\sqrt{3}i)}$$

$$= \frac{-2(1+i-\sqrt{3}i+\sqrt{3})}{(1^2+\sqrt{3}^2)} = \underline{\underline{-\frac{1}{2}\left(1+\sqrt{3}+i(1-\sqrt{3})\right)}}$$

$$\frac{(1+i)^5}{(1-i)^2} = \frac{-4(1+i)}{1-2i-1} = \frac{-4}{-2i}(1+i) = -2i(1+i) = \underline{\underline{2(1-i)}}$$

d) $$\frac{3+2i}{1-2i} = \frac{(3+2i)(1+2i)}{(1-2i)(1+2i)} = \frac{3+2i+6i-6}{5} = \underline{\underline{\frac{-3+8i}{5}}}$$

$$\frac{1+2i}{1-2i} = \frac{(1+2i)(1+2i)}{5} = \frac{1+4i-4}{5} = \underline{\underline{\frac{-3+4i}{5}}}$$

$$\frac{3-2i}{3+2i} = \frac{(3-2i)^2}{13} = \frac{9-12i-4}{13} = \underline{\underline{\frac{5-12i}{13}}}$$

$$\frac{(1+2i)^2}{3-2i} = \frac{(1+4i-4)(3+2i)}{13} = \frac{(-3+4i)(3+2i)}{13} = \frac{(-9+12i-6i-8)}{13} = \underline{\underline{\frac{-17+6i}{13}}}$$

$$\frac{1+2i}{(3-2i)^2} = \frac{1+2i}{5-12i} = \frac{(1+2i)(5+12i)}{169} = \frac{5+10i+12i-24}{169}$$

$$= \underline{\underline{\frac{19+22i}{169}}}$$

e)

$z_1+z_2 = \underline{6+4i}$

$z_1 \cdot z_2 = (1+2i)(5+2i) = 5+10i+2i-4 = \underline{\underline{1+12i}}$

$$\frac{z_2}{z_1} = \frac{(5+2i)(1-2i)}{5} = \frac{5+2i-10i+4}{5} = \underline{\underline{\frac{9-8i}{5}}}$$

$z_2^* z_1 = (z_1^* z_2)^* = (9-8i)^* = \underline{\underline{9+8i}}$

$$\frac{z_2}{z_1^*} = \frac{(5+2i)(1+2i)}{5} = \underline{\underline{\frac{1+12i}{5}}}$$