

Lösungen Blatt 2

1. a) $\cosh^2(x) = \frac{1}{4}(e^{2x} + 2 + e^{-2x})$
 $\sinh^2(x) = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$

$$\hookrightarrow \cosh^2(x) - \sinh^2(x) = \frac{1}{4}(e^{2x} - e^{2x} + e^{-2x} - e^{-2x} + 2 - (-2))$$

$$= 1$$

b) $\sinh(x+y) = \frac{1}{2}(e^{x+y} - e^{-x-y})$

Durch Bilden von $\sinh(x)\cosh(y)$ und $\sinh(y)\cosh(x)$ und Addieren, ergibt sich die gesuchte Form.

$$\sinh(x)\cosh(y) = \frac{1}{4}(e^{x+y} - e^{-x-y} + e^{x-y} + e^{-x+y})$$

$$\sinh(y)\cosh(x) = \frac{1}{4}(e^{x+y} - e^{-x-y} - e^{x-y} + e^{-x+y})$$

$$\hookrightarrow \sinh(x)\cosh(y) + \sinh(y)\cosh(x) = \frac{1}{2}(e^{x+y} - e^{-x-y})$$

$$= \sinh(x+y)$$

Analogie zu $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$.

c) $\cosh(x+y) = \frac{1}{2}(e^{x+y} + e^{-x-y})$

Bilden von $\cosh(x)\cosh(y)$ sowie $\sinh(x)\sinh(y)$

führt zum Ziel:

$$\cosh(x)\cosh(y) = \frac{1}{4}(e^{x+y} + e^{-x-y} + e^{x-y} + e^{-x+y})$$

$$\sinh(x)\sinh(y) = \frac{1}{4}(e^{x+y} + e^{-x-y} - e^{x-y} - e^{-x+y})$$

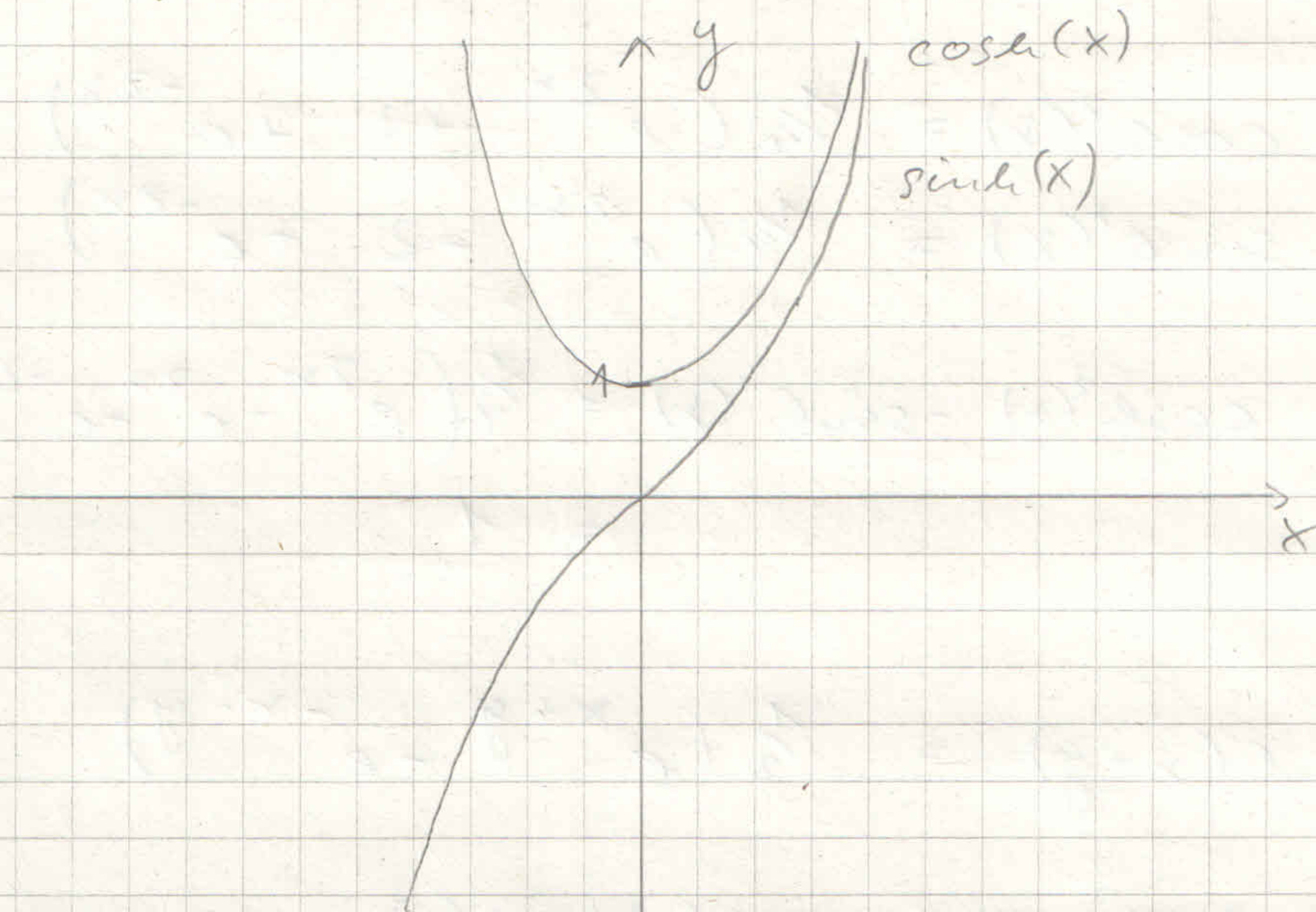
$$\hookrightarrow \cosh(x)\cosh(y) + \sinh(x)\sinh(y) = \frac{1}{2}(e^{x+y} + e^{-x-y})$$

$$= \cosh(x+y)$$

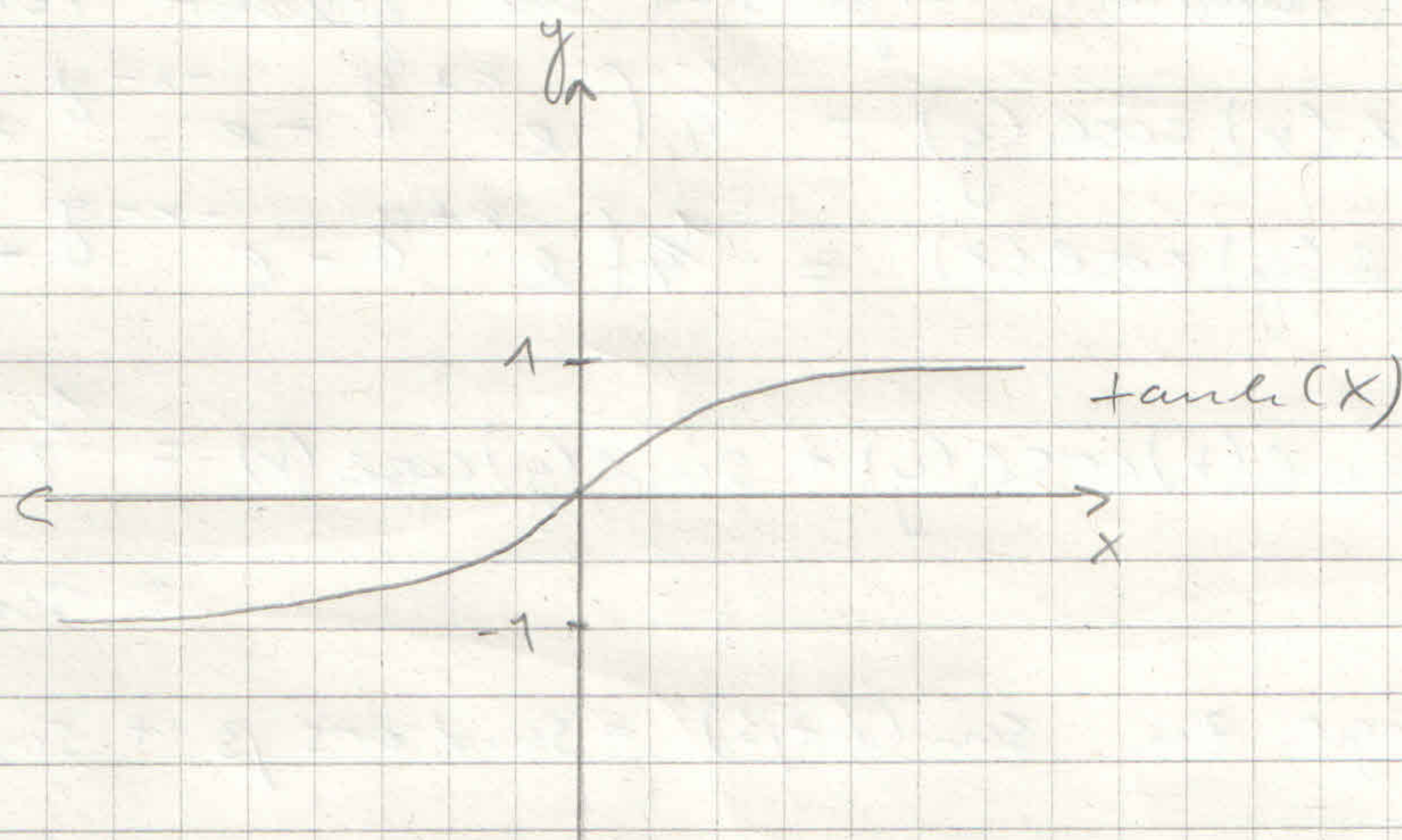
Unterschied zu $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

2.

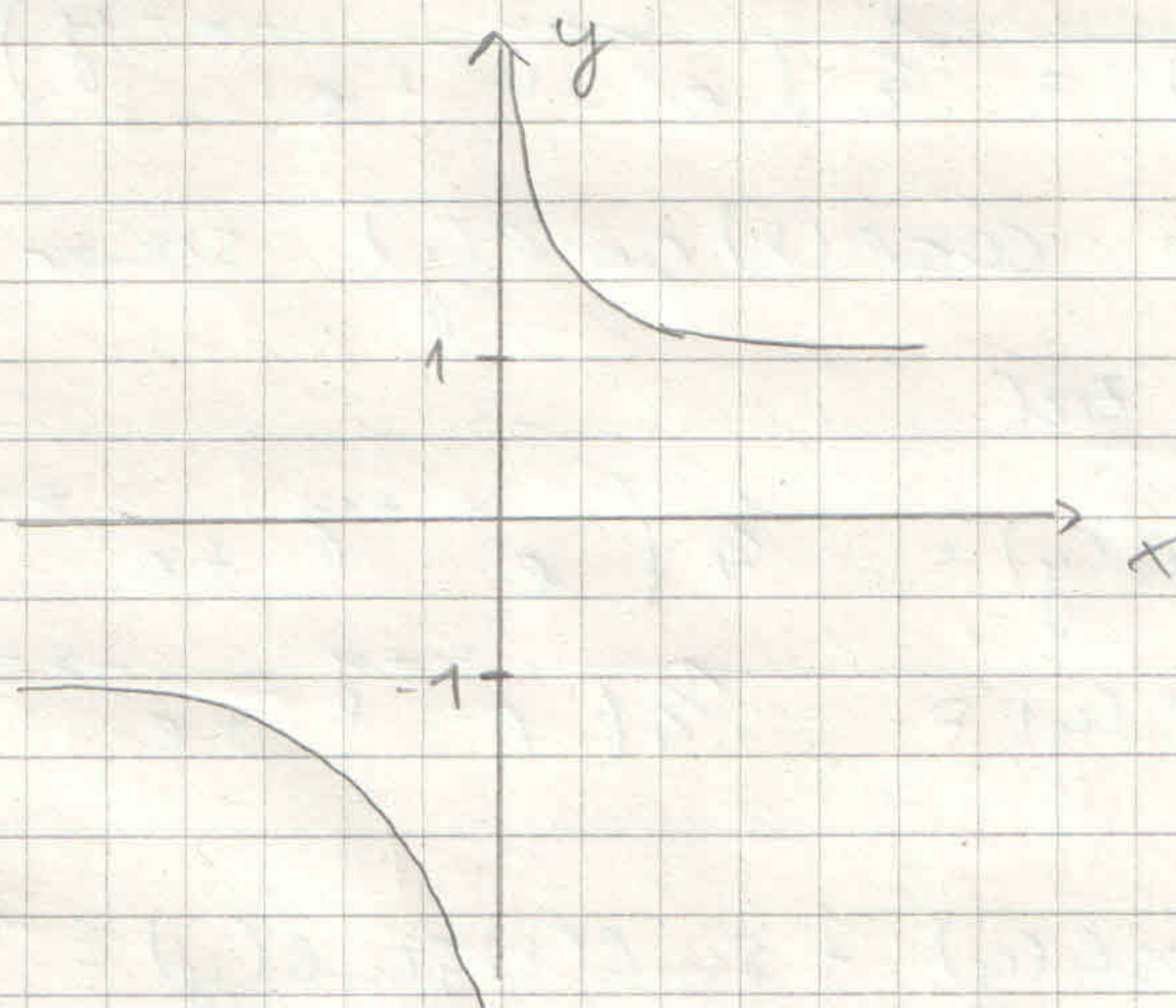
$\cosh(x)$, $\sinh(x)$



$\tanh(x)$



$\coth(x)$



3.

$$\log_a x = y$$

$$\Leftrightarrow a^y = x$$

$$\ln(a^y) = \ln(x)$$

$$y \cdot \ln(a) = \ln(x)$$

$$y = \frac{\ln(x)}{\ln(a)} = \log_a x$$

$$4. a) f'(x) = 12x^3 - 12x^2 + 2x - 2$$

$$b) f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$c) f'(x) = \sin x + x \cos x$$

$$d) f'(x) = \ln x + x \cdot \frac{1}{x} - 1 = \ln(x)$$

$$e) f'(x) = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$$

$$f) f'(x) = \frac{-\cos(x)}{\sin^2(x)}$$

$$5. a) f'(x) = \cosh(x)$$

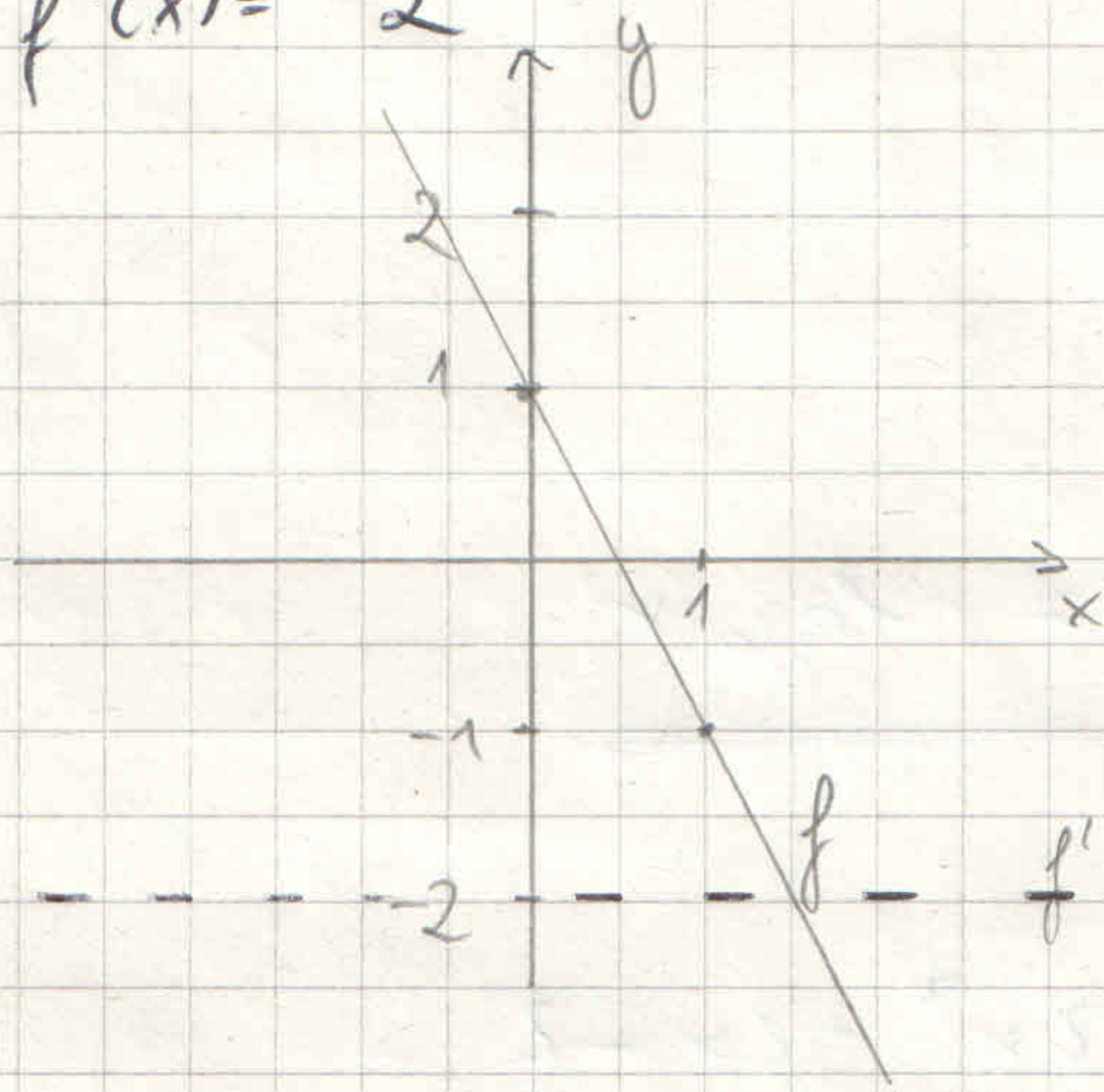
$$b) f'(x) = \sinh(x)$$

$$c) f'(x) = \frac{\cosh^2(x) - \sinh^2(x)}{\cosh^2(x)} = 1 - \tanh^2(x) = \frac{1}{\cosh^2(x)}$$

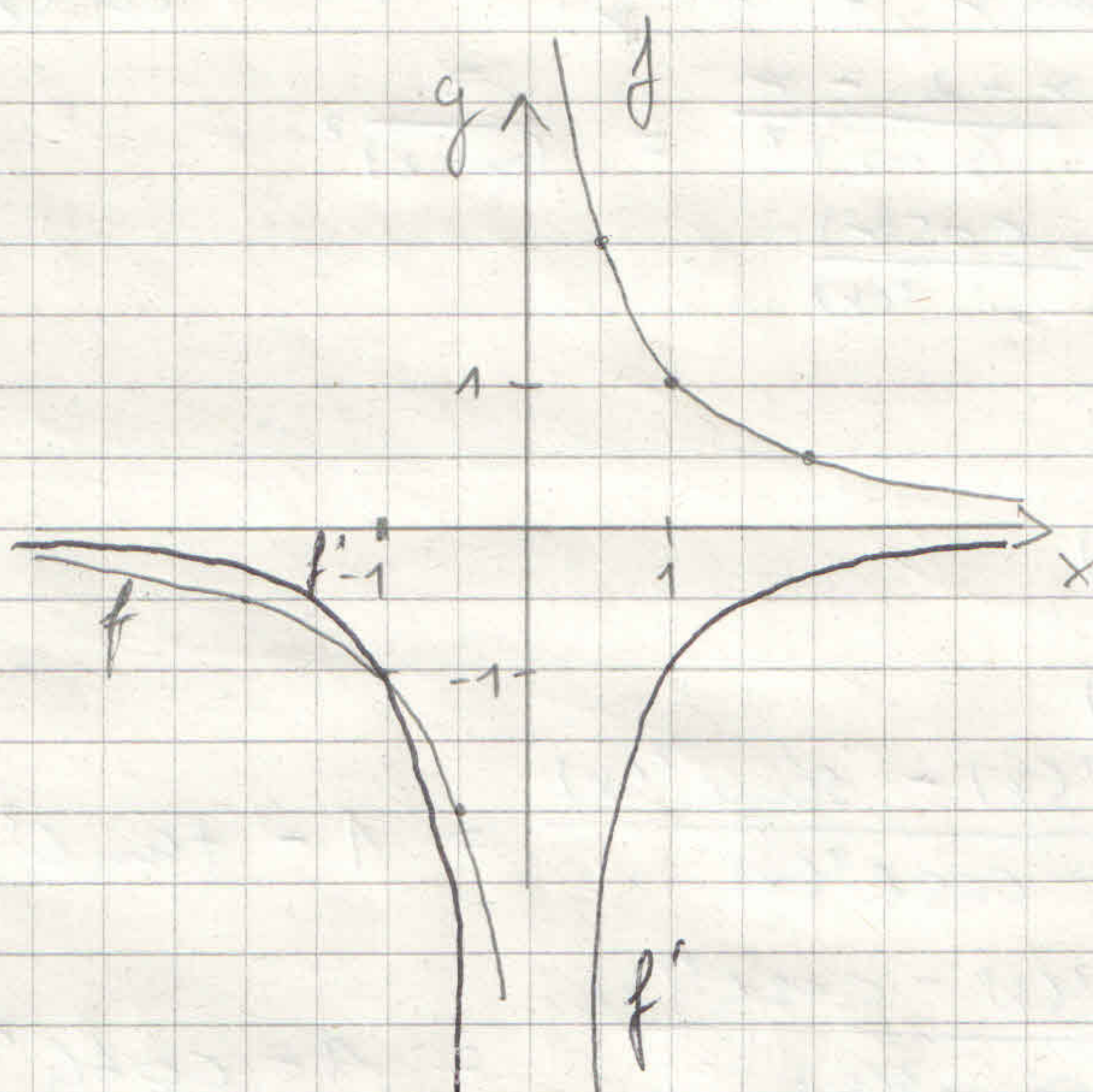
$$d) f'(x) = \frac{\sinh^2(x) - \cosh^2(x)}{\sinh^2(x)} = 1 - \coth^2(x) = \frac{-1}{\sinh^2(x)}$$

6. a) $f(x) = 1 - 2x$

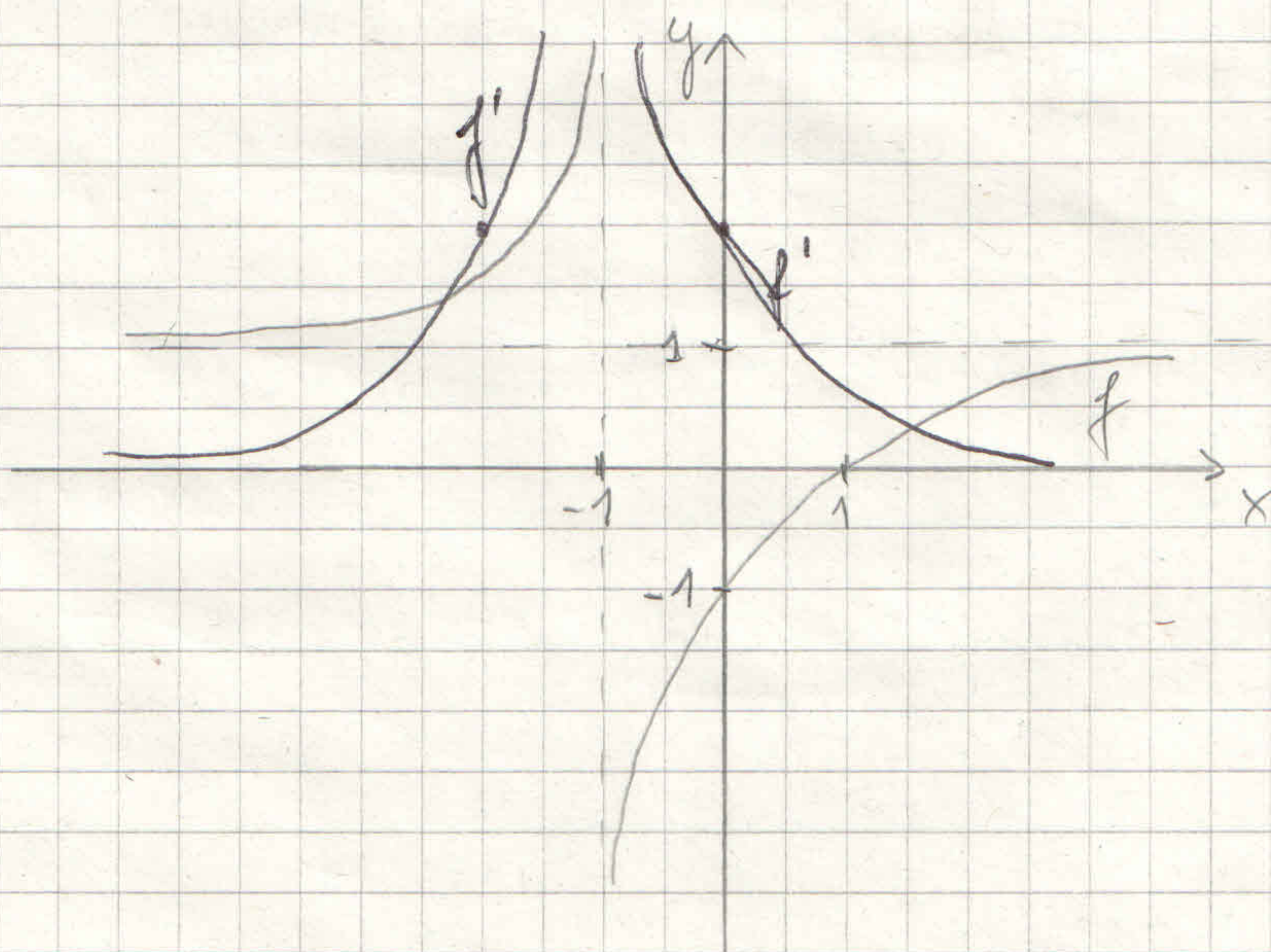
$f'(x) = -2$



b) $f(x) = \frac{1}{x}$, $f'(x) = \frac{-1}{x^2}$



c) $f(x) = \frac{x-1}{x+1}$, $f'(x) = \frac{2}{(x+1)^2}$



$$7. e) f(x) = \frac{1}{x^2 + a^2} ; f'(x) = \frac{-2x}{(x^2 + a^2)^2} ; f''(x) = \frac{6x^2 - 2a^2}{(x^2 + a^2)^3}$$

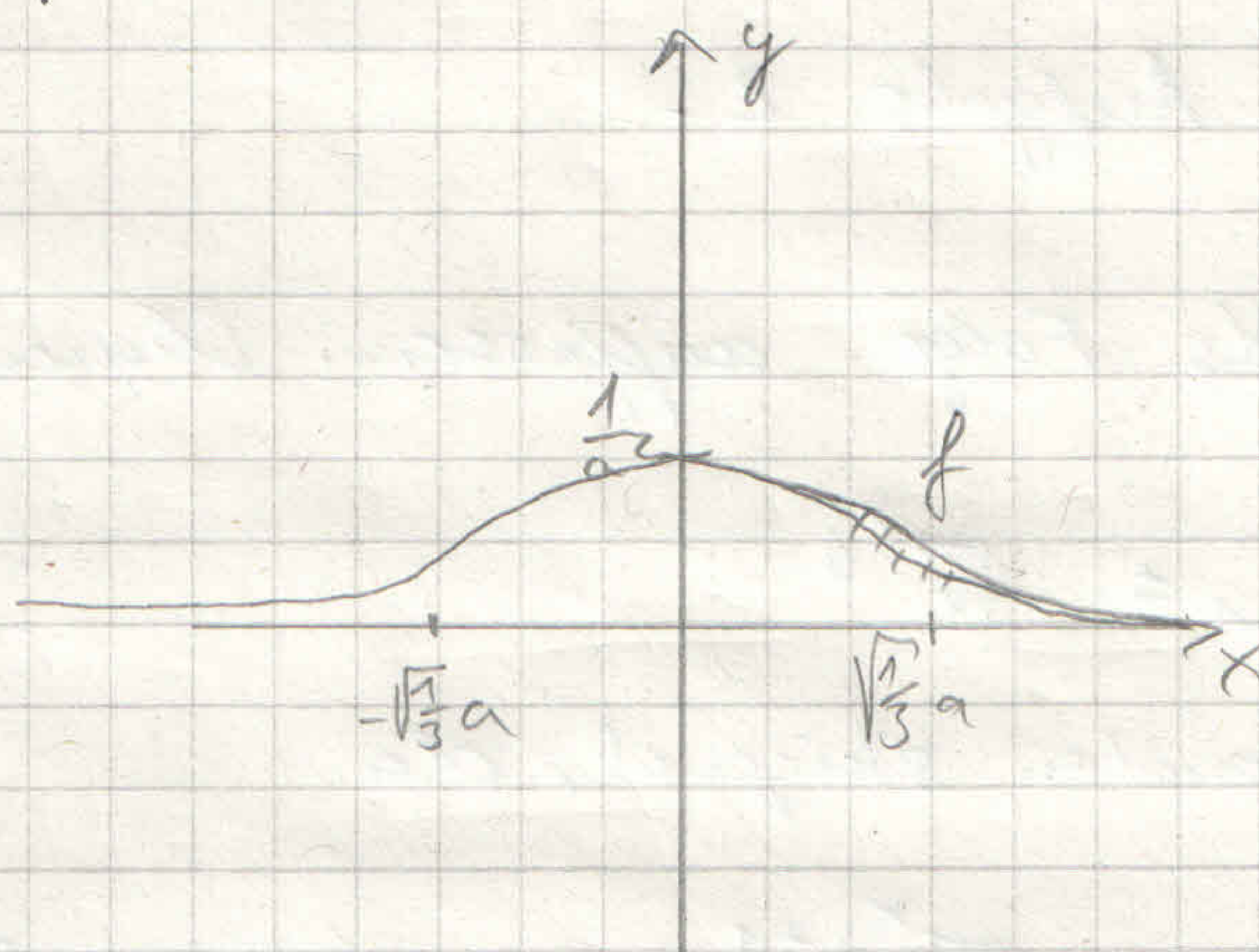
Wendepunkte:

$$f''(x) = 0$$

$$0 = 6x^2 - 2a^2$$

$$\Leftrightarrow x^2 = \frac{1}{3}a^2$$

$$x_{1,2} = \pm \sqrt{\frac{1}{3}}a$$



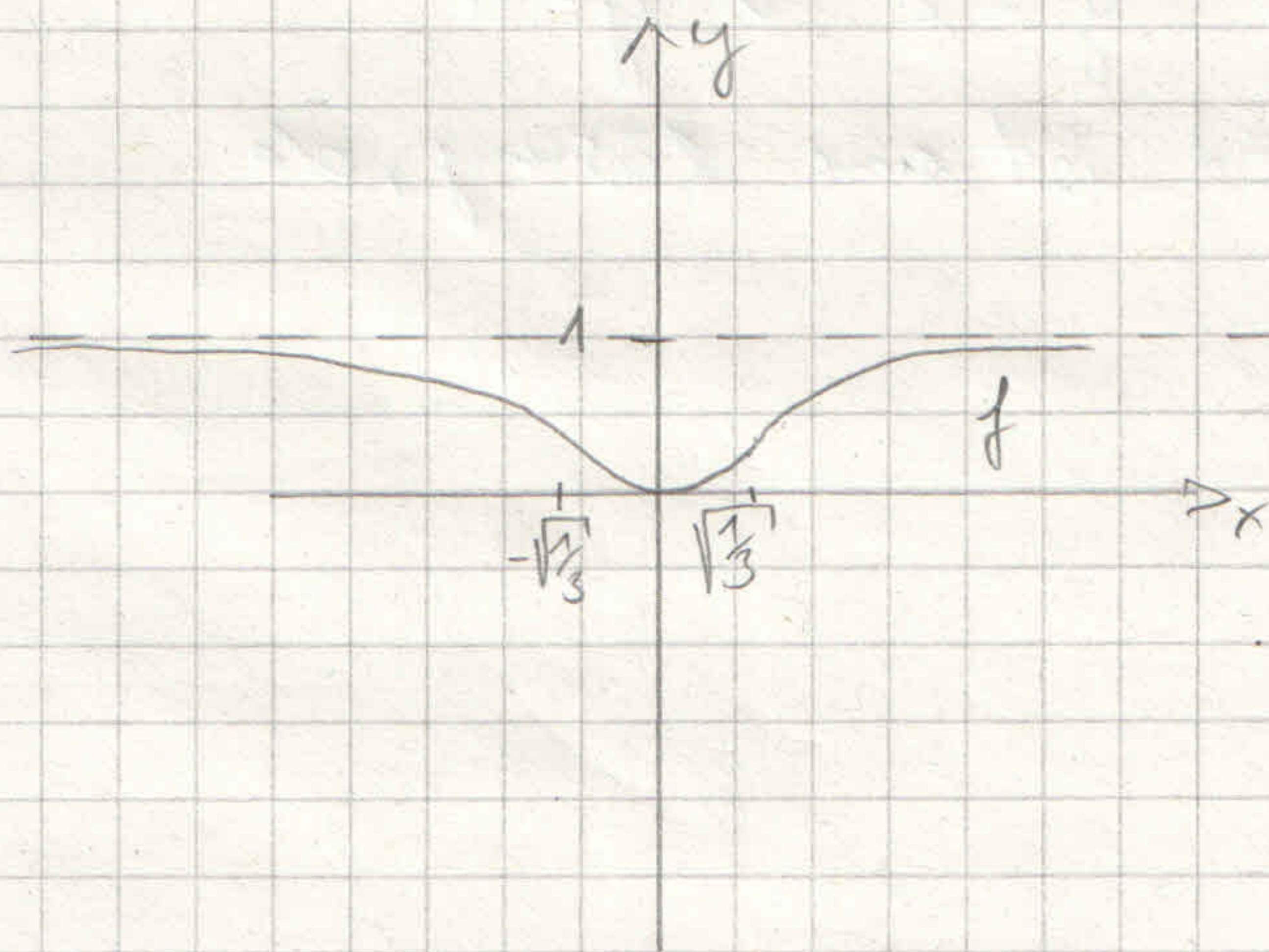
$$b) f(x) = \frac{x^2}{x^2 + 1} ; f'(x) = \frac{2x}{(x^2 + 1)^2} ; f''(x) = \frac{2 - 6x^2}{(x^2 + 1)^3}$$

$$f''(x) = 0$$

$$0 = 2 - 6x^2$$

$$x^2 = \frac{1}{3}$$

$$x_{1,2} = \pm \sqrt{\frac{1}{3}}$$



7. c) $f(x) = \tanh(x)$; $f'(x) = 1 - \tanh^2(x)$

$$f''(x) = -2 \tanh(x) (1 - \tanh^2(x))$$

Wendepunkte:

$$f''(x) = 0$$

$$0 = -2 \tanh(x)$$

$$\Rightarrow x = 0$$

$$\vee 0 = 1 - \tanh^2(x)$$

$$\tanh(x) \neq 1$$

Skizze: siehe Aufgabe 2.

8. Ansatz: Als Folge auffassen. Elemente x_j mit

$$x_{j+1} = \sqrt{2}^{x_j}$$

Streng monoton steigend, da

$$x_{j+2} = \sqrt{2}^{x_{j+1}} > x_{j+1} = \sqrt{2}^{x_j}$$

Folge konvergiert gegen den Wert 2

$$\Rightarrow y = \sqrt{2}^y$$

$$\log y = y \log \sqrt{2}$$

Lässt sich graphisch lösen.

$$\text{Lösungen: } y = 2 \vee y = 4$$

Nur $y = 2$ ist eine Lösung, da